



中山大學  
SUN YAT-SEN UNIVERSITY



国家超级计算广州中心  
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

# Compilation Principle 编译原理

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## 第7讲：语法分析(4)

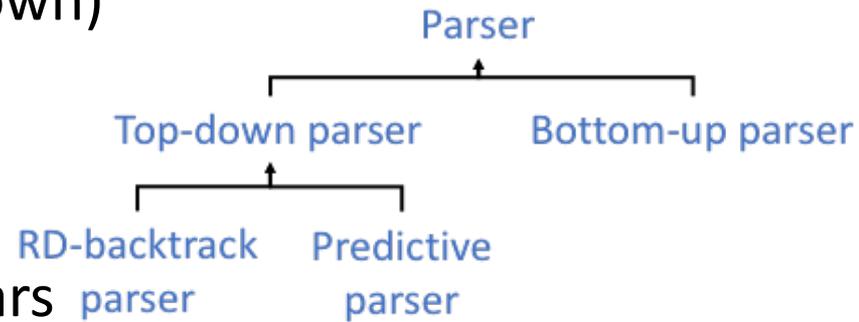
张献伟

[xianweiz.github.io](http://xianweiz.github.io)

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# Bottom-up Parsing[自底向上]

- Begins at leaves and works to the top
  - Bottom-up: **reduces**[归约] input string to start symbol
  - In the opposite direction from top-down
    - Top-down: expands start symbol to input string
  - In reverse order of rightmost derivation (In effect, builds tree from left to right, just like top-down)



- More powerful than top down
  - Don't need left factored grammars
  - Can handle left recursion
  - Can express a larger set of languages
  - And just as efficient

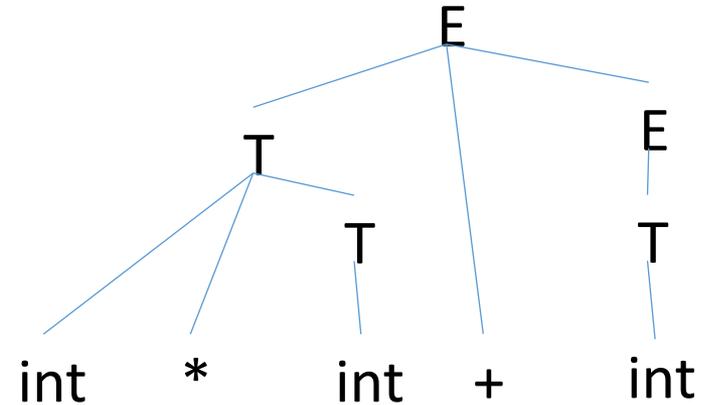
# Example

- Grammar

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

- String:  $\text{int} * \text{int} + \text{int}$



- The rightmost derivation of the parse tree

$$- E \Rightarrow T + E \Rightarrow T + T \Rightarrow T + \text{int} \Rightarrow \text{int} * T + \text{int} \Rightarrow \text{int} * \text{int} + \text{int}$$

- To recognize the string via bottom-up parsing

$$- \text{int} * \text{int} + \text{int} \Rightarrow \text{int} * T + \text{int} \Rightarrow T + \text{int} \Rightarrow T + T \Rightarrow T + E \Rightarrow E$$

# Bottom-up: Overview

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- An important fact:
  - Let  $\alpha\beta\omega$  be a step of a bottom-up parse
  - Assume the next reduction is by  $X \rightarrow \beta$
  - Then  $\omega$  is a string of terminals [i.e., 句子]
- **Why?**  $\alpha X\omega \rightarrow \alpha\beta\omega$  is a step in a rightmost derivation
- **Idea:** split string into two substrings
  - Right substring is as yet unexamined by parsing (a string of terminals)
  - Left substring has terminals and non-terminals
- The dividing point is marked by a **#**
  - The **#** is not part of the string
  - Initially, all input is unexamined  $\#x_1x_2 \dots x_n$

# Bottom-up: Shift-Reduce[移入-归约]

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- Bottom-up parsing is also known as **Shift-Reduce** parsing
  - Involves two types of operations: shift and reduce
- **Shift**: move # one place to the right
  - Shifts a terminal to the left string  
 $ABC\#xyz \Rightarrow ABCx\#yz$
- **Reduce**: apply an inverse production at the right end of the left string
  - If  $E \rightarrow Cx$  is a production, then  
 $ABCx\#yz \Rightarrow ABE\#yz$

# The Example

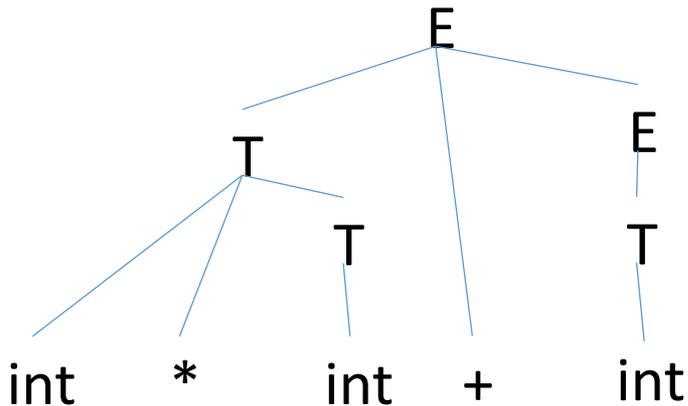
- Grammar

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

- String

int \* int + int



Sentential form	Operation
#int * int + int	Shift
int# * int + int	Shift
int * #int + int	Shift
int * int # + int	Reduce $T \rightarrow \text{int}$
int * T # + int	Reduce $T \rightarrow \text{int} * T$
T # + int	Shift
T + # int	Shift
T + int #	Shift
T + T #	Reduce $T \rightarrow \text{int}$
T + E #	Reduce $E \rightarrow T$
E #	Reduce $E \rightarrow T + E$

# Stack[棧]

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- Left string can be implemented by a **stack**
  - Top of the stack is the **#**
- **Shift** pushes a terminal on the stack
- **Reduce** does the following:
  - pops zero or more symbols off of the stack
    - production rhs
  - pushes a non-terminal on the stack
    - production lhs

# Key Issue[一个关键问题]

- How to decide when to shift or reduce?

- Example grammar:

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

- Consider the step  $\text{int} \# * \text{int} + \text{int}$

- We could reduce by  $T \rightarrow \text{int}$  giving  $T \# * \text{int} + \text{int}$

- **A fatal mistake:** no way to reduce to the start symbol E

Sentential form	Operation
$\# \text{int} * \text{int} + \text{int}$	Shift
$\text{int} \# * \text{int} + \text{int}$	Reduce $T \rightarrow \text{int}$
$T \# * \text{int} + \text{int}$	Shift
... ..	

- Intuition: want to reduce only if the result can still be reduced to the start symbol

# Handle[句柄]

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- Informally:
  - RHS of a production rule that, when reduced to LHS, will lead to the start symbol
- Definition: let  $\alpha\beta\omega$  be a sentential form where:
  - $\alpha, \beta$  is a string of terminals and non-terminals (yet to be derived)
  - $\omega$  is a string of terminals (already derived)
  - Then  $\beta$  is a **handle** of  $\alpha\beta\omega$  if:
    - $S \Rightarrow^* \alpha X \omega \Rightarrow \alpha\beta\omega$  by a rightmost derivation (apply rule  $X \rightarrow \beta$ )
- We only want to reduce at handles, and there is exactly one handle per sentential form
  - But where to find it?

# Handle: Example

- Grammar

$E \rightarrow T+E|T$

$T \rightarrow \text{int}*T | \text{int} | (E)$

- String

$\text{int} * \text{int} + \text{int}$

Step	Operation
#int * int + int	Shift
int# * int + int	Shift
int * #int + int	Shift
int * <b>int</b> # + int	Reduce $T \rightarrow \text{int}$
<b>int</b> * <b>T</b> # + int	Reduce $T \rightarrow \text{int}*T$
T # + int	Shift
T + # int	Shift
T + <b>int</b> #	Shift
T + <b>T</b> #	Reduce $T \rightarrow \text{int}$
<b>T</b> + <b>E</b> #	Reduce $E \rightarrow T$
E #	Reduce $E \rightarrow T+E$

# Handle Always Occurs at Stack Top

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- Why can't a handle occur on right side of #?
  - It can
  - But handle will eventually be shifted in, placing it at top of stack
  - In  $\text{int} * \# \text{int} + \text{int} \Rightarrow \text{int} * \text{int} \# + \text{int}$ ,  $\text{int}$  is eventually shifted to the top
- Why can't a handle occur on left side of #, i.e., in middle of the stack?
  - Can  $\text{int} * \text{int} + \# \text{int}$  occur? No.
  - Means parser shifted when it could have reduced when the handle was on top
  - If parser eagerly reduces when handle is at top of stack, never occurs
- Makes life easier for parser (need only access top of stack)

# Viable Prefix[活前缀]

- In shift-reduce parsing, the stack contents are always a viable prefix
  - A prefix of some right-sentential form that ends no further right than the end of the handle of that right-sentential form.
  - The handle is the substring that was introduced in the last step of rightmost derivation of that sentential form.
- 定义：一个可行前缀是一个最右句型的前缀，并且它没有越过该最右句型的最右句柄的右端
  - 举例：  $S \Rightarrow bBa \Rightarrow bbAa$ ，这里句柄是  $bA$ ，因此可行前缀包括  $bA$  的所有前缀（包括  $b, bb, bbA$ ），但不能是  $bbAa$ （因为越过了句柄）。

令  $G$  是一个文法,  $S$  是文法的开始符号, 假定  $\alpha\beta\delta$  是文法  $G$  的一个句型, 如果有

$$S \xRightarrow{*} \alpha A \delta \text{ 且 } A \xRightarrow{*} \beta$$

则称  $\beta$  是句型  $\alpha\beta\delta$  相对于非终结符  $A$  的短语。特别是, 如果有

$$A \Rightarrow \beta$$

则称  $\beta$  是句型  $\alpha\beta\delta$  相对于规则  $A \rightarrow \beta$  的直接短语, 一个句型的最左直接短语称为该句型的句柄。

# Ambiguous Grammars[二义文法]

- Conflicts arise with ambiguous grammars
  - Bottom up parsing predicts action w/ lookahead (just like LL)
  - If there are multiple correct actions, parse table will have conflicts
- Example:
  - Consider the ambiguous grammar  $E \rightarrow E * E \mid E + E \mid ( E ) \mid \text{int}$

Sentential form	Actions	Sentential form	Actions
int * int + int	shift	int * int + int	shift
...	...	...	...
E * E # + int	<b>reduce E → E * E</b>	E * E # + int	<b>shift</b>
E # + int	shift	E * E + # int	shift
E + # int	shift	E * E + int #	reduce E → int
E + int #	reduce E → int	E * E + E #	reduce E → E + E
E + E #	reduce E → E + E	E * E #	reduce E → E * E
E #		E #	

# Ambiguous Grammars (cont.)

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- In the red step shown, can either shift or reduce by  $E \rightarrow E * E$ 
  - Both okay since precedence of  $+$  and  $*$  not specified in grammar
  - Same problem with associativity of  $+$  and  $*$
- As usual, remove conflicts due to ambiguity ...
  - 1. Rewrite grammar/parser to encode precedence and associativity
    - Rewriting grammar results in more convoluted grammars
    - Parser tools have other means to encode precedence and association
  - 2. Get rid of remaining ambiguity (e.g. if-then-else)
    - No choice but to modify grammar
- Is ambiguity the only source of conflicts?
  - Limitations in lookahead-based prediction can cause conflicts
  - But these cases are very rare

# Properties of Bottom-up Parsing

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- Handles always appear at the **top of the stack**
  - Never in middle of stack
  - Justifies use of stack in shift – reduce parsing
- Results in an easily generalized **shift – reduce** strategy
  - If there is no handle at the top of the stack, shift
  - If there is a handle, reduce to the non-terminal
  - Easy to automate the synthesis of the parser using a table
- Can have conflicts
  - If it is legal to either shift or reduce then there is a shift-reduce conflict
  - If there are two legal reductions, then there is a reduce-reduce conflict
  - Most often occur because of ambiguous grammars
    - In rare cases, because of non-ambiguous grammars not amenable to parser